Assignment for colloquium 4.

# Stability analysis for a piecewise linear system

Consider the system in Example 1 of

M. Johansson and A. Rantzer: Computation of Piecewise Quadratic Lyapunov Functions for Hybrid Systems, IEEE Transactions on Automatic Control, Vol. 43, No. 4, 1998, pp. 555 – 559

The system is given by:

where the regions are defined by , with

see also the top part of Figure 3 in Johansson and Rantzer. Note that all regions contain the origin.

We want to estimate the speed of convergence by finding the PWQ Lyapunov function which maximizes α in

subject to the Lyapunov function being continuous

and each component of the Lyapunov function being positive in its domain of definition

See lecture notes or Johansson and Rantzer for further details on .

Note that the equation above involving α is bilinear in α and , and including this would lead to a non-convex problem. Instead of attempting to find a solution using specialized non-linear solvers, we instead try to solve the problem for gradually increasing (positive) values of α.

Maximize α, and find the corresponding ’s.

Note, you may use the ’s proposed by Johansson and Rantzer, but these are not unique. I don’t expect the choice of these to affect the optimal value for α.